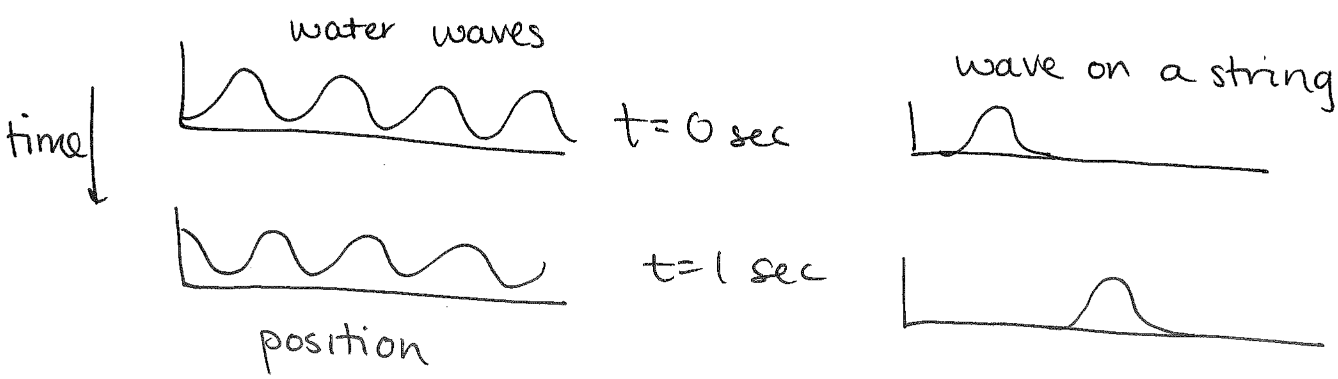
Differential Equations - short version

We've seen a lot of waves today, and we will be using what we learned today in the rest of the course. In general, we'd like to know how to describe these phenomena mathematically.

We saw that traveling waves change in both position and in time

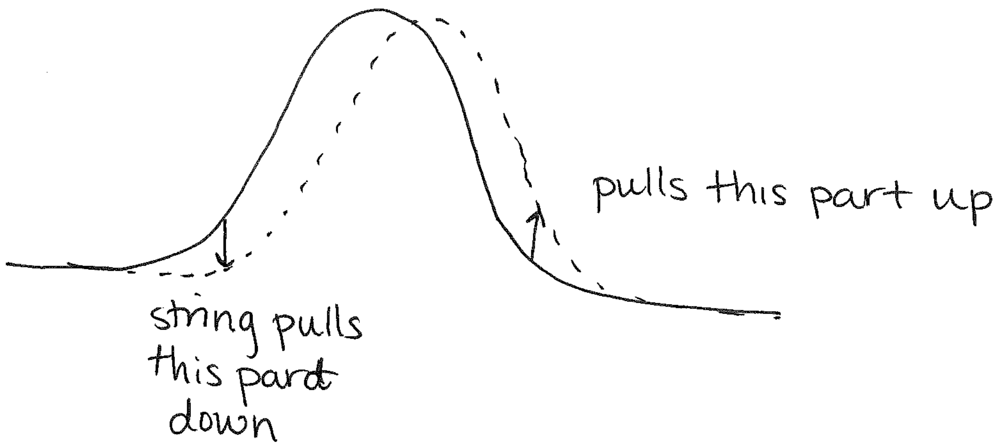


What is something interesting / noticeable about the way these waves travel? ... Brainstorm...

→ the shape of the wave didn't change
→ the wave didn't spread out

This may seem obvious, but this is crucial to describing/understanding traveling waves of this type

Let's think about the wave on a string. What makes the wave move forward?



parts of the string exert <u>forces</u> on other parts of string.
What do forces do? From Newton's Laws?

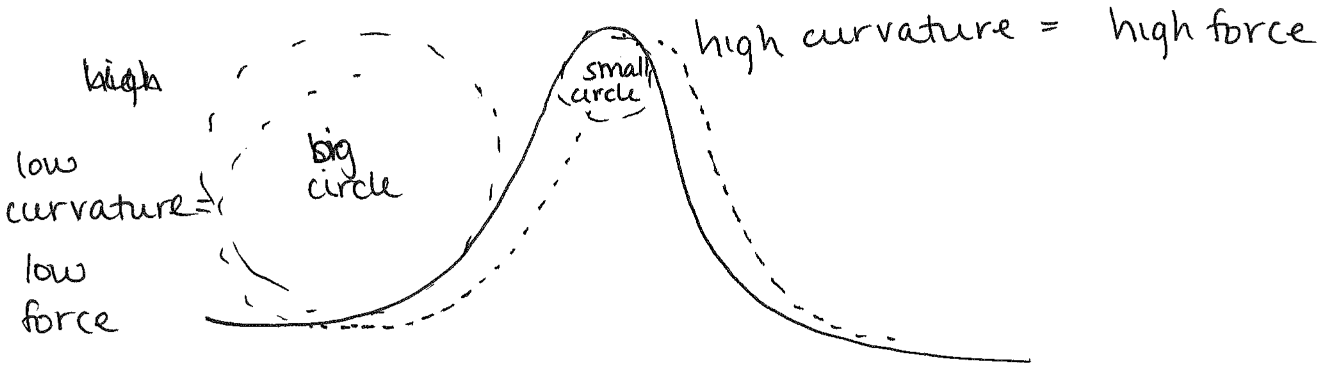
force ⟷ acceleration

$$F = m \cdot a$$

force of engine makes car accelerate ...

This law tells us / governs what is happening with our wave.

Now it turns out that the force (on one part of string from another) is directly related to curvature:



force $\longleftrightarrow$ ~~m~~ acceleration

||

curvature $\longleftrightarrow$ acceleration

~~has to do w/ changes in position~~

has to do with:

changes in height with respect to position

~~has to do with changes in time~~

has to do with:

changes in height with respect to time.

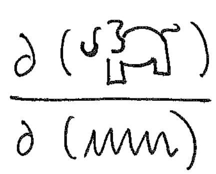
Specifically:

change in change in height with respect to position $\longleftrightarrow$ change in change in height with respect to time

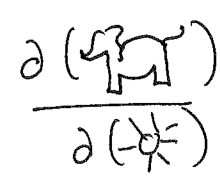
This is called the wave equation

$\longrightarrow$ describes waves that do not spread out

To write this mathematically:



= change in elephants with respect to grass

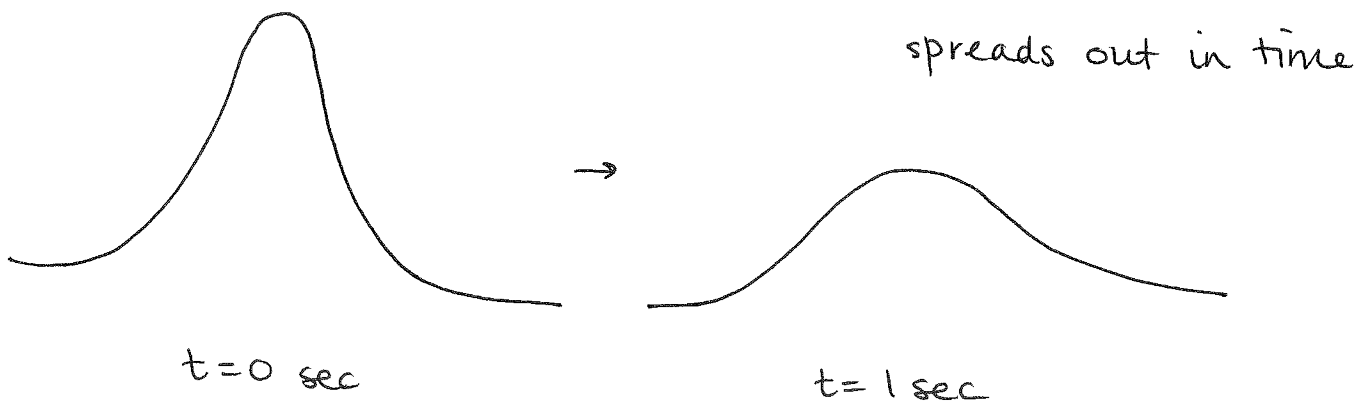


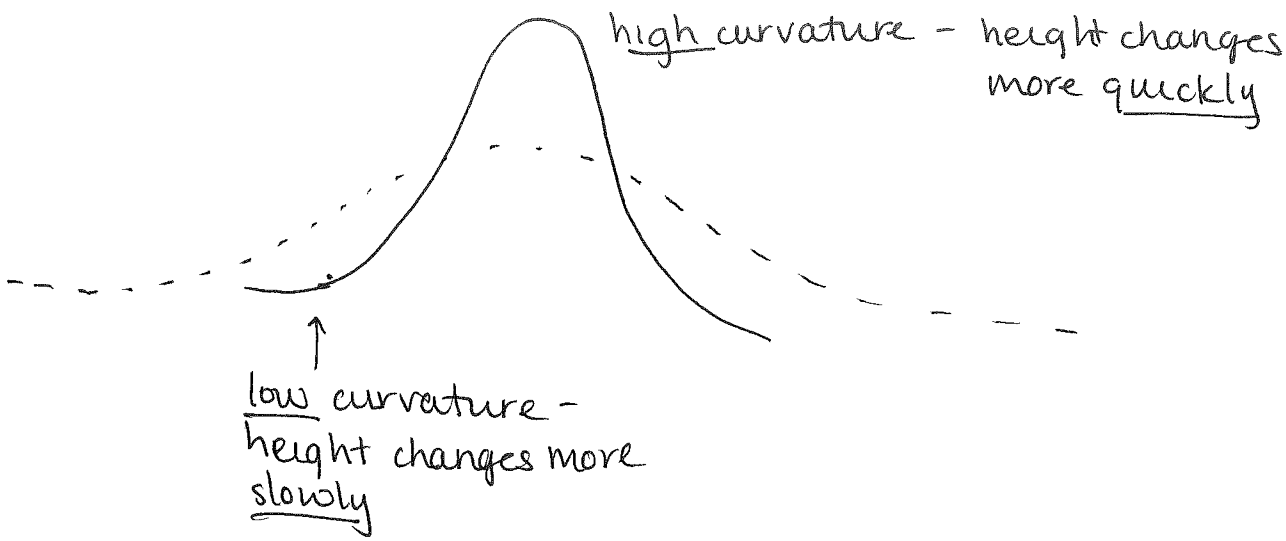
$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) \overset{\propto}{\longleftrightarrow} \frac{\partial}{\partial t}\left(\frac{\partial h}{\partial t}\right)$$

$$\frac{\partial^2 h}{\partial x^2} \longleftrightarrow \frac{\partial^2 h}{\partial t^2}$$

There's one additional equation that will come in handy and is very similar (at first glance) to the wave equation … the diffusion equation

What is diffusion? What happens?





curvature ⟷ velocity

change in change in height with respect to position ⟷ change in ~~change~~ height with respect to time

Diffusion equation - describes "wave" that spreads out in time